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CALCULATED PROFILES OF H I LINES OF INTEREST FOR SOLAR PLASMA ELECTRIC FIELD MEASUREMENTS

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Abstract. We present calculated Stark-polarized line profiles for a number of H I lines observed in the visible and infrared emission spectrum of solar prominences and other limb activity. For use in measurements of possible electric fields in these structures, we also calculate curves giving the difference in line width between the $\frac{1}{2}(I \pm Q)$ profiles as a function of electric-field intensity. Our calculations take into account magnetic fields in these structures, and incorporate typical observed values of Doppler broadening. These calculations explicitly consider the H I fine structure neglected in previous work, and thus are more accurate in the range of low to intermediate electric-field intensity likely to be encountered in solar plasmas ($E < 10^3$ V cm^{-1}). Our results enable us to compare behavior when **E** and **B** are parallel, or perpendicular. We draw particular attention to the high electric-field sensitivity of the transitions between high levels such as 12-8 and 15-9 in H I, observed in prominences at wavelengths around 11μ. Their sensitivity is roughly an order of magnitude larger than that of the high Paschen-series lines used in solar plasma electric field studies so far.

1. Introduction

The behavior of the hydrogen atom under the influence of quasi-steady electric and magnetic fields is of considerable interest in diagnostic studies of laboratory plasmas (e.g., Nguyen-Hoe, Drawin, and Herman, 1967; Berezin, Bubovoi, and Lyublin, 1972a, b; Moore, Davis, and Gottscho, 1984; Levinton *et al.*, 1990), of stellar atmospheres (e.g., Underhill and Waddell, 1959; Mathys, 1983, 1984), and of solar activity (e.g., Dravins, 1973; Foukal and Hinata, 1991).

Underhill and Waddell provided comprehensive tables of splittings and oscillator strengths of the components of hydrogen lines for the linear Stark effect, covering the Lyman, Balmer, Paschen, and Brackett series up to the principal quantum number $n = 18$. (We note in passing a misprint in the tabulated data of the 18–3 transition of the Paschen series (P18) given by Underhill and Waddell: the component listed at the displacement $X = 204$ is missing while the component with $X = 240$ is quoted twice.) Applying the Holtsmark theory of pressure broadening (e.g., Griem, 1974), they could also provide tabulated values of the Stark-broadening function for each line, which enable one to calculate the line absorption coefficient. However, no magnetic field was considered in their calculations, and the fine structure of hydrogen was also neglected. In particular, this last assumption limits the applicability of Underhill and Waddell results to

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the case of relatively high-intensity fields, according to the line considered (e.g., $E \gg 10^3$ V cm^{-1} for Hα; $E \gg 5$ V cm^{-1} for P18). On the other hand, the strong-field regime suffers the limitations of the n-mixing phenomenon (see Section 3), which is neglected in the Underhill and Waddell calculations.

In order to overcome the difficulties of the Holtsmark theory, a more extended study, aimed at reproducing the Stark-broadened profiles of lower hydrogen lines such as Lα, Lβ, and Hα, was performed by Nguyen-Hoe, Drawin, and Herman, who adopted the impact theory for pressure broadening (e.g., Griem, 1974). They also considered the presence of a stationary magnetic field of arbitrary orientation, so they had to follow a perturbation approach to the problem, limiting the calculation to first order. However, the authors chose to neglect the contribution of the fine structure. A different approach to the same problem was followed by Mathys (1983, 1984), who applied the unified classical path theory of Smith, Cooper, and Vidal (1969) to overcome the limits of the impact theory, somewhat improving the results of Nguyen-Hoe, Drawin, and Herman. However, the fine structure was still neglected in the work of Mathys.

Finally, we point out a relevant review by Braun (1993) on the problem of Rydberg atoms in external fields. The highly excited states of Rydberg atoms allow a semi-classical approach (WKB method) to the problem, safely neglecting the fine-structure contribution. The main features of the hydrogen spectrum, such as splittings of the levels and oscillator strengths, are investigated in the case of crossed electric and magnetic fields, also accounting for the quadratic Stark and Zeeman effects. The results of this work might be useful for investigating hydrogen lines involving very high Bohr levels ($n > 20$).

However, for the lowest transitions, and particularly in the case of low electron-density plasmas, the contribution of the fine structure should be considered, since its neglect is responsible for a large part of the discrepancies between the calculated and observed Stark profiles (e.g., Ehrich and Kelleher, 1978).

Another reason for including fine structure is the problem of possible residual degeneracy of the hydrogen levels in the first-order perturbation theory for particular configurations of the external fields. It is known (e.g., Solov'ev, 1983) that in the absence of fine structure three such configurations exist, namely, the cases of pure Stark effect ($B = 0$), of pure Zeeman effect ($E = 0$), and of perpendicular magnetic and electric fields. In such cases, a second-order perturbation expansion is advisable, especially if one considers that for some values of the parabolic quantum numbers the first-order corrections may vanish (e.g., Braun, 1993). But if the fine structure is included in the perturbation Hamiltonian, the degeneracy is completely removed to first order if a magnetic field is present, while a residual degeneracy still occurs in the case of pure Stark effect. In this scheme the second-order corrections are far less essential in determining the main features of the energy pattern of a Bohr level, so they can be safely neglected, if we are not interested in dealing with very strong fields. For Rydberg atoms, however, the fine structure is completely negligible, and one also gets quite rapidly into the strong-field regime. In such

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case, the second-order corrections can severely affect the structure of a Bohr level (Braun, 1993).

The perturbation approach to the problem of the hydrogen atom in the presence of stationary electric and magnetic fields of arbitrary orientation is also followed in our work to first order, but the contribution of the fine structure is now included. The domain of applicability then extends to the cases of low to intermediate electric-field intensity (i.e., $E < 10^3$ V cm^{-1}, according to the line considered), which are typical of solar-physics investigations. However, the strong-field case can be treated as well in our approach. For instance, by omitting the contribution of the fine structure, we could recover the results of Underhill and Waddell for the linear Stark effect.

We focus on several H I lines of practical interest in remote sensing of electric fields in solar plasmas, using the polarization structure of their Stark components. Hα (λ6563 Å) was first used for this purpose in flare observations on the disk by Dravins (1973), in an attempt to detect the high electric-field intensities predicted in the discharge theory of flares put forward by Alfvén and Carlqvist (1967). This line is optically thick in most solar structures. We use our results to estimate the electric-field sensitivity of the line in flares on the limb and on the disc.

The high Balmer and Paschen series lines B10, B18, P10, P18, at $\lambda\lambda$3798, 3692, 9015, 8438 Å, respectively, have been used in observations of prominences and other limb activity using a spectro-polarimeter 'electrograph' installed at Sacramento Peak Observatory (Moran and Foukal, 1991; Foukal and Behr, 1994). Finally, we are now also extending the electrograph technique to high-series transitions around λ 11μ (Zirker, 1985) observed to have relatively favorable emission intensity in prominences.

2. Assumptions of the Model

The results of the calculations we present in this paper are limited in their applicability by the two following assumptions:

(a) We only consider the case of optically thin emission lines. This assumption avoids the need to solve the vector radiative-transfer equation, since the components of the Stokes vector (I, Q, U, V) become linear functions of the polarized emission coefficients ϵ_i ($i = 0, 1, 2, 3$, identifying the four Stokes parameters I, Q, U, V; see Landi Degl'Innocenti, 1983), which in turn are proportional to the (scalar) source function.

(b) The atomic polarization of the initial level of the transition can be neglected. Therefore, the density matrix for that level, which enters the expression of the coefficients ϵ_i, is simply proportional to the unit matrix.

The absence of atomic polarization is a typical feature of lines which are formed under the restrictive hypothesis of the LTE regime. More generally, it can be considered a reasonable approximation in low-pressure plasmas, where the atomic-

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level populations are mainly determined by isotropic recombination processes. This is true, in particular, for highly excited initial levels (Landi Degl'Innocenti, 1993).

Given the above assumptions, our calculations are not applicable to absorption lines, expecially if saturation effects are present. Also, the effects of atomic polarization may need to be considered for lower-level transitions in some solar structures, such as prominences which are illuminated by a strongly anisotropic radiation field. In these cases, a self-consistent solution of the radiative-transfer equation and of the statistical-equilibrium equations is required, which can be obtained in practice only for simple geometries of the plasma and of the radiation field, and under simplifying hypotheses about the atomic structure (e.g., Landi Degl'Innocenti, Bommier, and Sahal-Bréchot, 1990, 1991). But our calculations should significantly improve upon present estimates of electric-field sensitivity in the optically-thin transitions from upper levels of H I which are of greatest practical interest in current studies of electric fields in solar plasmas (Foukal and Hinata, 1991; Foukal and Behr, 1994).

3. Procedure

The polarization profiles of a line formed in a transition between two Bohr levels n and m are in general obtained as the solution of the vector radiative-transfer equation, which requires the previous computation of some relevant coefficients determining the emission and absorption properties of the line (Landi Degl'Innocenti, 1983). In our calculation for an optically thin line, only the polarized emission coefficients are needed, according to assumption (a). To compute them, we specify the eigenstates (i.e., eigenvalues and eigenvectors) of all the sublevels which constitute the two Bohr levels of the transition as well as the orientation of the line of sight. In general, the density matrix of the initial level of the transition is also needed, but in our calculations this is simply proportional to the unit matrix, according to assumption (b).

The additional hypothesis that the n-mixing phenomenon (i.e., the mixing of states from different Bohr levels) could be neglected is necessary, if we intend to attack the problem of the hydrogen atom subject to external fields by means of perturbation methods. Following Casini and Landi Degl'Innocenti (1993), the unperturbed hydrogen atom is described by Schrödinger's Hamiltonian for the electron in the Coulomb potential of the nucleus, while the perturbation Hamiltonian accounts for the fine structure and for the external fields. The diagonalization of the perturbation matrix for a level n then yields the first-order corrections to the energies and the zero-order eigenvectors of the atomic sublevels.

It is worth noting that the simultaneous diagonalization of the fine-structure and external-fields Hamiltonians enables one to recover the results typical of the intermediate-field cases, such as the incomplete Paschen–Back effect (e.g., Bethe

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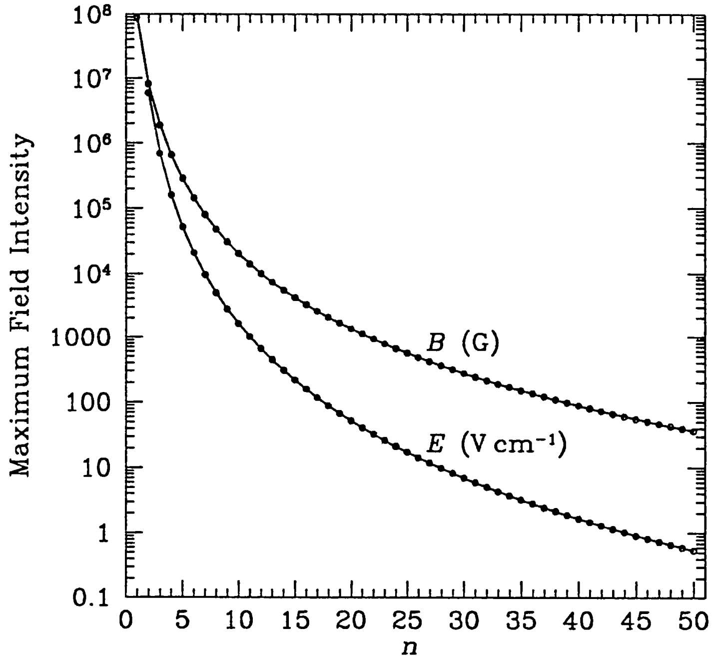

Fig. 1. The maximum electric- and magnetic-field intensities, E and B, which can be treated perturbatively for a given Bohr level, n, plotted against n.

and Salpeter, 1957). These results cannot be reproduced within the LS-coupling approximation, which is usually adopted in this kind of calculations, because it is not suited to the intermediate- to strong-field regime. Of course, the weak- and strong-field cases are treatable as well, in our scheme. However, the assumption that the n-mixing phenomenon could be neglected sets an upper limit to the range of intensities of the external fields which can be considered in a perturbation calculation of a hydrogen line, according to the Bohr levels of the transition. For higher intensities, two adjacent Bohr levels would overlap because of their dispersion, and one should also take into account the n-mixing phenomenon. (For highly excited levels, for which the classical perturbation theory works fairly well, the neglect of the n-mixing phenomenon is far less critical; e.g., Braun, 1993.) Thus, if $\delta W_{E,B}$ is the dispersion of the level due to either the electric or the magnetic field, and ΔW_n is the energy interval between the levels n and $n+1$, it is required that the condition $\delta W_{E,B} < \epsilon \Delta W_n$ be satisfied for sufficiently small ϵ. In Figure 1, the limits for the electric and magnetic fields, determined by the above condition when $\epsilon = 0.1$, are plotted against the Bohr level, n.

Once the corrected eigenstates for the two levels of the transition have been found, they must be used to compute the polarized emission coefficients ϵ_i, i.e., the theoretical profiles of the optically-thin line considered.

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4. Optimization of Electric-Field Sensitivity

To choose optimal lines for detection of solar-plasma electric fields, it is important to also ascertain whether the Zeeman sensitivity of a line can affect the Stark-effect observations. It is well known that a measure of the sensitivity of a line to a magnetic field of intensity B is given by the effective Landé factor g_{eff}. This quantity enters the expression for the separation Δ_V between the centers of gravity of the circular-polarization signals $\mathcal{P}_V^{\pm} = \frac{1}{2}(I \pm V)$. In the frequency domain, it is

$$\Delta_V = 2\frac{\mu_0}{\hbar} g_{\text{eff}} B_{\parallel} , \tag{1}$$

where μ_0 is the Bohr magneton and $B_{\parallel}$ is the component of the magnetic field along the line of sight. Under the assumptions of the present work, it has been demonstrated (Casini and Landi Degl'Innocenti, 1994a) that for any hydrogen line $g_{\text{eff}} = 1$, so that the Zeeman sensitivity in the frequency domain, as estimated by Equation (1), is the same for all hydrogen lines. However, we cannot compare the Stark and Zeeman sensitivities of a hydrogen line through the investigation of the circular-polarization profiles, as the electric field does not contribute to the quantity Δ_V. Nor can the separations Δ_Q (or Δ_U) between the centers of gravity of the linear-polarization signals $\mathcal{P}_Q^{\pm}$ (or $\mathcal{P}_U^{\pm}$) be used under our assumptions, as they are shown to vanish identically (in the frequency domain) in the presence of either electric or magnetic fields. Therefore, no comparison can be made between the Stark and Zeeman sensitivities of hydrogen lines, as long as we limit ourselves to considering only the first-order moments of the polarization profiles (which are proportional to the Δ's).

One must then introduce the second-order moments of the polarization profiles, by considering the standard deviations of the linear-polarization signals. In the wavelength domain, these are given by

$$\sigma_{Q,U}^{\pm} = \sqrt{\langle \lambda_{\pm}^2 \rangle_{Q,U} - \langle \lambda_{\pm} \rangle_{Q,U}^2} \, . \tag{2}$$

Then, one calculates the dependence of the observable modulations, $\Delta\sigma = |\sigma^+ - \sigma^-|$, upon the electric-field intensity (e.g., Moran and Foukal, 1991), as an objective way of estimating the Stark sensitivity of a line.

In Equation (2),

$$\langle \lambda_{\pm}^q \rangle_S = \frac{\int \mathcal{P}_S^{\pm}(\lambda)\,(\lambda - \overline{\lambda})^q \, \mathrm{d}\lambda}{\int \mathcal{P}_S^{\pm}(\lambda)\, \mathrm{d}\lambda} \tag{3}$$

are the qth-order wavelength moments, with respect to $\overline{\lambda}$, of the linear-polarization signals $\mathcal{P}_S^{\pm} = \frac{1}{2}(I \pm S)$ ($S = Q$, U). In our calculations, $\overline{\lambda}$ is the center of gravity of the intensity profile in the absence of external fields.

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Recent theoretical calculations (Casini and Landi Degl'Innocenti, 1994b) show that the contribution of the magnetic field to the second-order frequency moments of the polarization profile is also independent of the line considered. Therefore, only the criterion of high Stark sensitivity is relevant to the choice of the optimal lines for electric-field investigations.

A simple estimate of the Stark sensitivity can be obtained, to first order, from the energy dispersion of a Bohr level, n, due to the linear Stark effect, through the quantity (e.g., Bethe and Salpeter, 1957)

$$\delta W_E(n) = 3e_0 a_0 E n(n-1) \,. \tag{4}$$

Here, e_0 and a_0 are, respectively, the absolute value of the electronic charge and the radius of the first Bohr orbit, and E is the electric field intensity. One can then estimate the dependence of the total energy dispersion of a transition $n-m$ on the Bohr levels by

$$\delta W_E(n,\, m) \approx \sqrt{[\delta W_E(n)]^2 + [\delta W_E(m)]^2} \sim \sqrt{n^2(n-1)^2 + m^2(m-1)^2} \,, \tag{5}$$

which suggests that it is advisable to choose transitions which have high values of both n and m. Exact calculations of the electric-field contribution to the second-order moments of the polarization profiles (Casini, 1995) show that a more adequate estimate is

$$\delta W_E(n,\, m) \sim \sqrt{n^2(n^2-1) + m^2(m^2-1) - 2n^2m^2} \,. \tag{6}$$

Further theoretical investigations on this topic are planned, however, in order to define an optimum Stark-sensitivity parameter for the hydrogen lines.

5. Results

In all the results reported here, we assumed the simplest case of both electric and magnetic fields orthogonal to the line of sight. In addition the electric and magnetic fields are always assumed to be parallel to each other, as this is the most likely configuration of observable, quasi-steady macroscopic electric fields in solar plasmas (Foukal and Hinata, 1991). However, the case of orthogonal fields is worth studying as well, since in the presence of a magnetic field a motional Stark effect must be present in any thermal plasma, due to the microscopic motional electric field experienced by each atom (e.g., Moran and Foukal, 1991). This microscopic electric field would be anisotropic in the case of a directed magnetic field, and thus would be detectable by measurement of the linear-polarization signals $\mathcal{P}_Q^{\pm}$, as discussed here. Similar motional electric fields have been studied in the laboratory

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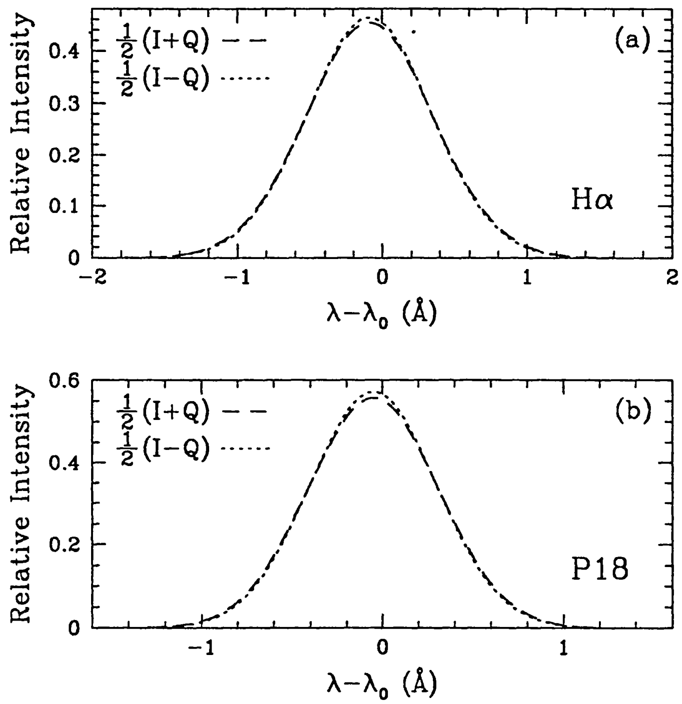

Fig. 2. The normalized linear-polarization signals, $\mathcal{P}_Q^{\pm}$, of (a) Hα for parallel electric and magnetic fields, $E = 1000$ V cm^{-1}, $B = 100$ G, convolved with a Doppler profile with FWHM = 1 Å; (b) P18 transition for $E = 10$ V cm^{-1}, $B = 10$ G, and FWHM = 0.8 Å. Here λ_0 is the unperturbed wavelength of the transition.

using ion beams (e.g., Rosenbluh *et al.*, 1978; Levinton *et al.*, 1990). The case of perpendicular fields is discussed at the end of this section.

We calculated the thermal-broadened profiles $\mathcal{P}_Q^{\pm}$ of some hydrogen emission lines in the optically thin case. This required, according to the procedure outlined in Section 3, the calculation of the emission coefficients ϵ_I and ϵ_Q for each line component (a transition between two Bohr levels n and m has $4n^2m^2$ fine-structure components). For such calculation we used Equations (19) in Casini and Landi Degl'Innocenti (1993). The (normalized) profiles $\mathcal{P}_Q^{\pm}$ are then obtained as

$$\mathcal{P}_Q^{\pm}(\lambda) = \frac{\sum_k \frac{1}{2}[\epsilon_I(\lambda_k) \pm \epsilon_Q(\lambda_k)]\Phi(\lambda - \lambda_k)}{\sum_k \epsilon_I(\lambda_k)}, \tag{7}$$

where $\Phi(\lambda)$ is the gaussian profile determined by the thermal-broadening mechanisms, and the summations are extended to all the line components. Typical results are shown in Figures 2 and 3.

We also calculated, under the same assumption of optically-thin emission lines, several curves of the linear-polarization modulation against the electric-field intensity, using Equations (2) and (3). The needed formulae to calculate the quantities $\langle\lambda_{\pm}\rangle_Q$ and $\langle\lambda_{\pm}^2\rangle_Q$, in the absence of other broadening mechanisms than the

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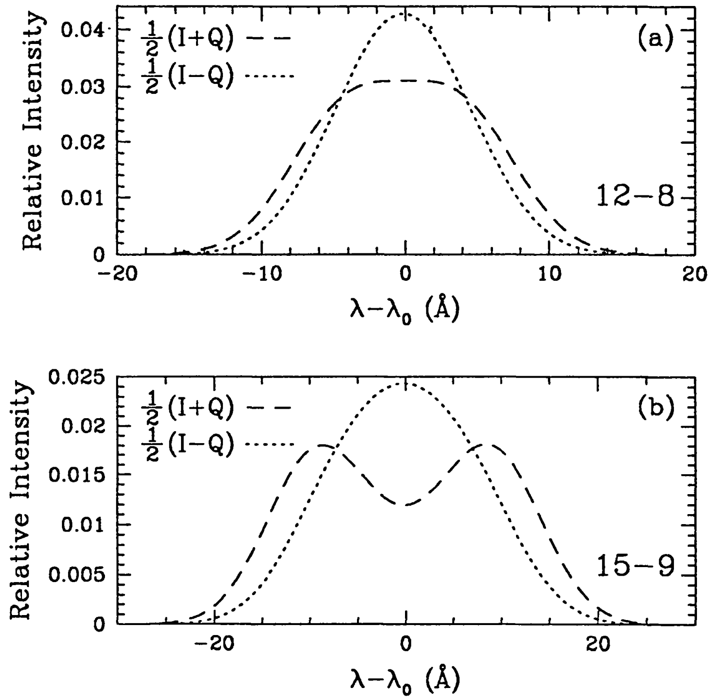

Fig. 3. The normalized linear-polarization signals, $\mathcal{P}_Q^{\pm}$, of (a) the 12–8 transition with $E = 10$ V cm^{-1}, $B = 10$ G, and FWHM = 8.7 Å; (b) the 15–9 transition with $E = 10$ V cm^{-1}, $B = 10$ G, and FWHM = 11 Å.

external fields, have been given (in the frequency domain) by Casini and Landi Degl'Innocenti (1994a, b). (However, if we ignore the quadratic Zeeman effect, the contribution of $\langle\lambda^{\pm}\rangle_Q$ to Equation (2) can be safely neglected; cf., Casini and Landi Degl'Innocenti, 1994a.) The extension of Equation (2) to the case of broadened profiles has also been discussed by Casini and Landi Degl'Innocenti (1994b, Section 5). Typical examples of such modulation curves, with different broadening profiles $\Phi(\lambda)$, are shown in Figures 4–6.

The shape of the $\mathcal{P}_Q^{\pm}$ profiles for Hα is shown in Figure 2(a). The dependence of the observed modulation on the electric-field intensity is given in Figure 4, for magnetic fields of 100 G (corresponding to flare or post-flare loop structures in the low corona), and for a range of Doppler widths corresponding to typical values for post-flare loops and flares. Flare-like parameters are used in our Hα profile illustration, because only the highest electric-field intensities (most likely in flares) could produce detectable modulation in Hα. We see that detectable modulation amplitudes of order 10 mÅ require $E \sim 10^3$ V cm^{-1}, even for the smallest Doppler broadening, and for an optically-thin line. Modulation would be lower in the realistic (for Hα) case of $\tau > 1$.

Figure 2(b) shows similar polarized profiles for the P18 line, for $E = 10$ V cm^{-1}, $B = 10$ G and Doppler broadening of 0.8 Å. These parameters are typical of quiescent prominences, as observed by Moran and Foukal (1991). An electric field of 50 V cm^{-1} is sufficient to recover the symmetric pattern of the Stark effect in

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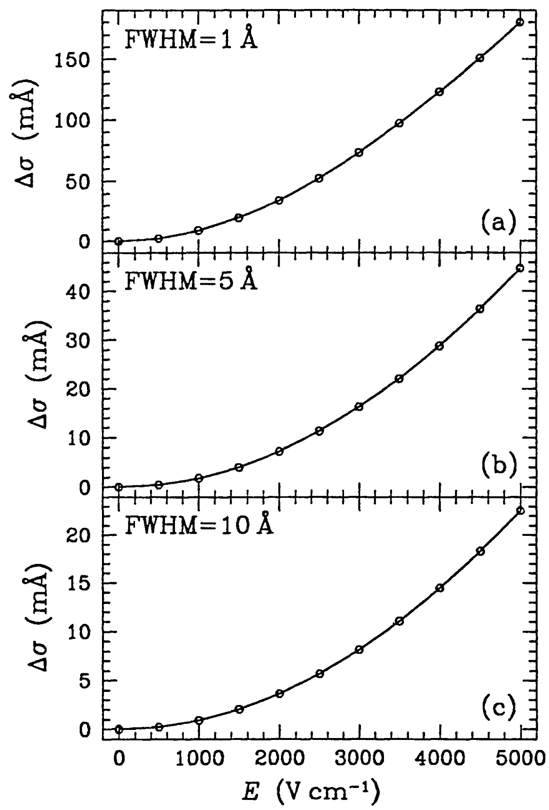

Fig. 4. The modulations, $\Delta\sigma$, plotted against the electric field, E, for Hα convolved with Doppler profiles of: (a) FWHM = 1 Å, (b) FWHM = 5 Å, (c) FWHM = 10 Å, and B = 100 G.

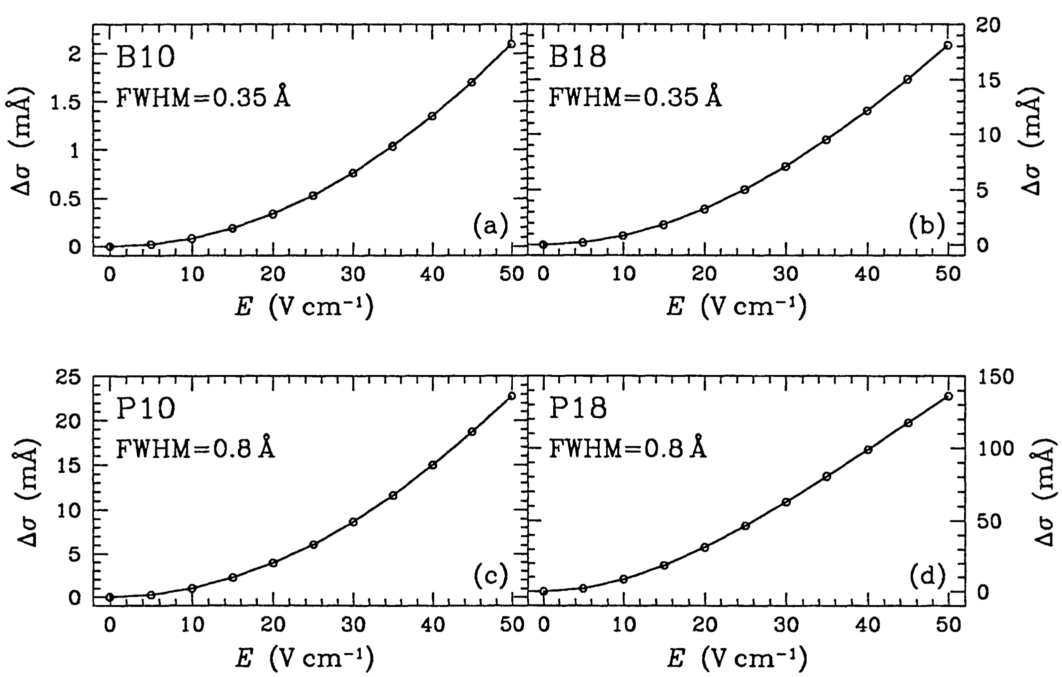

Fig. 5. The modulations, $\Delta\sigma$, plotted against the electric field, E, for the transitions: (a) B10, (b) B18, convolved with a Doppler profile with FWHM = 0.35 Å, and (c) P10, (d) P18, with FWHM = 0.8 Å. Magnetic fields parallel to the electric field, with intensities B = 10–100 G, are assumed.

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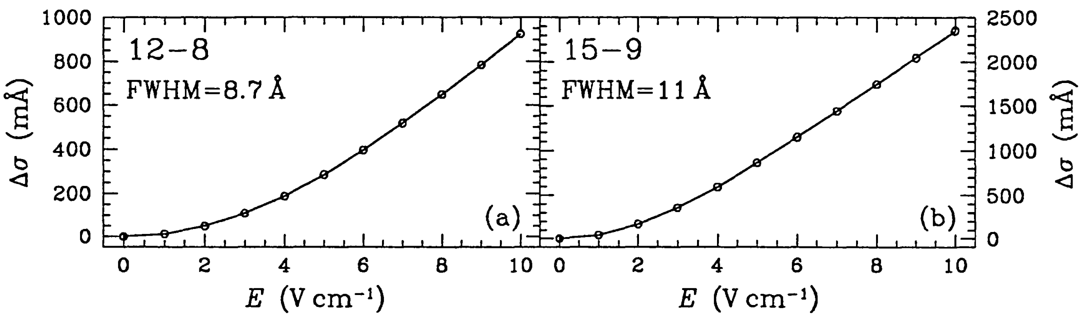

Fig. 6. The modulations, $\Delta\sigma$, plotted against the electric field, E, for the transitions: (a) 12–8, convolved with a Doppler profile with FWHM = 8.7 Å, (b) 15–9, convolved with a Doppler profile with FWHM = 11 Å. A magnetic field, $B = 10$ G, parallel to the electric field is assumed.

P18, and Figure 5(d) shows that a linear trend of the modulation amplitude against the electric-field intensity is reached at somewhat lower intensities. Figure 5(d) also shows that detectable modulations $\Delta\sigma > 10$ mÅ in P18 occur at $E > 10$ V cm^{-1}, with only weak dependence (not detectable on the plots) on the intensity of the magnetic field, between $B = 10$ G to $B = 100$ G. The sensitivity for P10 is much lower, as shown in Figure 5(c), with about 35 V cm^{-1} required to produce 10 mÅ modulation.

Figures 5(a) and 5(b) show similar plots for B10 and B18. Comparing the sensitivities of B18 and P18 when the two lines originate in a plasma of the same temperature and turbulent velocity distribution, we find that the P18 line is much more sensitive than B18. That is, even if we scale the measurable threshold of $\Delta\sigma$ in B18 to 4 mÅ to account for its narrower profile by the factor of 2.3, we still find that the electric-field values required to produce that modulation are about twice those required in P18.

The extremely high electric-field sensitivities of the 12–8 and 15–9 transitions of H I observed near 11μ can be seen already in the difference between the $\mathcal{P}_Q^+$ and $\mathcal{P}_Q^-$ profiles shown in Figure 3, for $E = 10$ V cm^{-1} and $B = 10$ G. Compared to the modulation in the high Balmer and Paschen lines, where it appears at the level below 1% of the total line width, the modulation in 15–9 for $E = 5$ V cm^{-1} appears at the 20% level! The modulation curve shown in Figure 6(b) shows that for a total FWHM of 11 Å (which is the observed width of this line in prominences; Zirker, 1985), and for $B = 10$ G, we would expect that equivalent modulation at the 1% of FWHM level (i.e., 100 mÅ) would yield an electric-field sensitivity of about 1 V cm^{-1}. This is an order of magnitude better than the modulation achieved in the P18 line.

We now return to the behavior in the case of **E** perpendicular to **B**. Theory indicates (Casini and Landi Degl'Innocenti, 1994b) that parallel and perpendicular fields produce almost identical modulations in the regime where the energy contribution of one field dominates the energy contributions of the other, and of the fine structure. This regime corresponds to the linear part of the modulation curves (cf.,

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Figures 4–6, where the electric field is dominant). The modulation curves in the two cases of parallel and perpendicular fields differ noticeably only if the energy contributions of the two fields have comparable values (for the line considered). Then, in the case of parallel fields, a value of the ratio E/B exists such that the corresponding modulation, $\Delta\sigma$, vanishes. This critical value is given by

$$\left(\frac{E}{B}\right)_{n-m}^{\mathrm{crit}} = \frac{\mu_0}{e_0 a_0} \frac{1}{\sqrt{3A_2(n,\, m)}} , \tag{8}$$

where $A_2(n,\, m)$ is a coefficient, obtained from theory, which is tabulated for all hydrogen lines up to the level $n = 50$ (Casini, 1995). For example, it is $A_2(8,\, 12) \approx 1.400 \times 10^3$ for the 12–8 transition, so that $(E/B)^{\mathrm{crit}}_{12-8} \approx 0.017$ V cm^{-1} G^{-1}. For magnetic fields up to about 100 G, the critical regime of electric-field intensity for this line always occurs below the present limit of detection. Thus, it is unlikely that measurements of significant electric fields would underestimate the true field intensity due to presence of significant magnetic fields.

6. Conclusions

Our more accurate calculations broadly confirm the Stark sensitivities of Balmer and Paschen line widths to electric-field intensities, which were previously estimated by Foukal, Little, and Gilliam (1988) and Moran and Foukal (1991), using the earlier results of Underhill and Waddell (1959), which neglected fine structure.

One interesting result of our calculations is the very high electric-field sensitivity of the high H I transitions such as 12–8 and 15–9, near 11μ. Given the high signal-to-noise ratio of the 12–8 transition in a prominence observed by Brault and Noyes (1983) (see Zirker, 1985), it appears that electric-field measurements should be feasible in prominences at the sensitivity level below 1 V cm^{-1}. This sensitivity is roughly a factor of 5 better than achieved with the electrograph operating on the P18 line at NSO/Sacramento Peak (Foukal and Behr, 1994).

Recent observations by one of us (PF) of the infrared spectrum of a prominence with the FTS at NSO/Kitt Peak, confirm the high intensity of the 12–8 transition, and the much lower, but measurable intensity of the 15–9 transition. Future observations will show whether this transition is also strong in post-flare loops, and limb flares, enabling more sensitive electrograph observations in those solar magnetic structures as well.

A second important result is the similarity of the calculated modulations, $\Delta\sigma$, found for $\mathbf{E}\|\mathbf{B}$ and $\mathbf{E} \perp \mathbf{B}$ (always transverse to the line of sight), when the ratio E/B is well outside the critical range given by Equation (8). This is contrary to the claim of Rosenbluh *et al.* (1978) that, for perpendicular fields, the hydrogen atom can respond to the electric field in second order only. The claim of Rosenbluh *et al.* disagrees with other references on the subject (e.g., Solov'ev, 1983; Braun, 1993), and also with our calculations in the limit of vanishing fine structure. This

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similarity of the calculated modulations ensures the sensitivity of electric field measurements to both parallel and perpendicular fields. That is an important consideration, since the perpendicular, motional electric field in prominences probably defines the 'minimum' electric field that must be present in the solar atmosphere, at an observable intensity level of roughly 0.5 V cm^{-1} (Moran and Foukal, 1991).

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References

Alfvén, H. and Carlqvist, P.: 1967, *Solar Phys.* **1**, 221.
Bethe, H. A. and Salpeter, E. E.: 1957, *Quantum Mechanics of One- and Two-Electron Atoms*, Springer-Verlag, Berlin.
Berezin, A., Bubovoi, L., and Lyublin, B.: 1972a, *Soviet Phys. Tech. Phys.* **16**, 2914.
Berezin, A., Bubovoi, L., and Lyublin, B.: 1972b, *Soviet Phys. Tech. Phys.* **17**, 750.
Brault, J. and Noyes, R.: 1983, *Astrophys. J.* **269**, L61.
Braun, P. A.: 1993, *Rev. Mod. Phys.* **65**, 115.
Casini, R.: 1995, *Astron. Astrophys. Suppl.*, in press.
Casini, R. and Landi Degl'Innocenti, E.: 1993, *Astron. Astrophys.* **276**, 289.
Casini, R. and Landi Degl'Innocenti, E.: 1994a, *Astron. Astrophys.* **291**, 668.
Casini, R. and Landi Degl'Innocenti, E.: 1994b, *Astron. Astrophys.*, in press.
Dravins, D.: 1973, *Astrophys. Letters* **13**, 243.
Ehrich, D. L. and Kelleher, D. E.: 1978, *Phys. Rev.* **A17**, 1686.
Foukal, P. V. and Behr, B. B.: 1994, in V. Rušin, P. Heinzel, and J.-C. Vial (eds.), *Proceedings of IAU Colloquium No. 144*, Veda Publishing, p. 177.
Foukal, P. and Hinata, S.: 1991, *Solar Phys.* **132**, 307.
Foukal, P., Little, R., and Gilliam, L.: 1988, *Solar Phys.* **114**, 65.
Griem, H.R.: 1974, *Spectral Line Broadening by Plasmas*, Academic Press, New York.
Landi Degl'Innocenti, E.: 1983, *Solar Phys.* **85**, 3.
Landi Degl'Innocenti, E.: 1993, private communication.
Landi Degl'Innocenti, E., Bommier, V., and Sahal-Bréchot, S.: 1990, *Astron. Astrophys.* **235**, 459.
Landi Degl'Innocenti, E., Bommier, V., and Sahal-Bréchot, S.: 1991, *Astron. Astrophys.* **244**, 391.
Levinton, F., Gammel, G., Kaita, R., Kugel, H., and Roberts, D.: 1990, *Rev. Sci. Instr.* **61**, 2914.
Mathys, G.: 1983, *Astron. Astrophys.* **125**, 13.
Mathys, G.: 1984, *Astron. Astrophys.* **139**, 196.
Moore, C., Davis, G., and Gottscho, R.: 1984, *Phys. Rev. Letters* **52**, 538.
Moran, T. and Foukal, P.: 1991, *Solar Phys.* **135**, 179.
Nguyen-Hoe, Drawin, H. W., and Herman, L.: 1967, *J. Quant. Spectr. Rad. Transfer* **7**, 429.
Rosenbluh, M., Panock, R., Lax, B., and Miller, T. A.: 1978, *Phys. Rev.* **A18**, 1103.
Smith, E. W., Cooper, J., and Vidal, C. R.: 1969, *Phys. Rev.* **185**, 140.
Solov'ev, E. A.: 1983, *Soviet Phys. JEPT* **58**, 63.
Underhill, A. B. and Waddell, J. H.: 1959, *Stark Broadening Functions for the Hydrogen Lines*, National Bureau of Standards, Circ. 603.
Zirker, J.: 1985, *Solar Phys.* **102**, 33.

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